# TWMA

جمعية التونسيات للرياضيات
**TUNISIAN WOMEN MATHEMATICIANS' ASSOCIATION**

# FÉLICITATIONS ET BRAVO!
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## De la Meilleure Thèse de Doctorat en Mathématiques Pures
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Titre : On m-isometric transformations

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# Abstract

This thesis is concerned with $(m, p)$-isometries defined on normed vector spaces and generalizations. The results of this thesis are in four papers:

[A] T. Bermúdez, A. Saddi and H. Zaway, *(A, m)-isometries on Hilbert spaces*, Linear Algebra Appl., **540** (2018), 95–111.

> **We study the relation between $(A, m)$-isometries and $m'$-isometries. We prove that the spectrum of an $(A, m)$-isometry with a non-zero operator $A$ must intersect the unit circle. Further, we give some dynamic properties of the class of $(A, m)$-isometries.**

[B] T. Bermúdez and H. Zaway, *On $(m, \infty)$-isometries: Examples*, Results Math., **74** (2019), no. 3, 74–108.

> **We study unilateral weighted shift operators which are $(m, \infty)$-isometries for some integers $m$. In particular, we prove that any power of $(2, \infty)$-isometry is also a $(2, \infty)$-isometry. Moreover, this result is not valid for $(3, \infty)$-isometry.**

[C] T. Bermúdez, A. Bonilla and H. Zaway, *$C_0$-semigroups of $m$-isometries on Hilbert spaces*, J. Math. Anal. Appl., **472** (2019), no. 2, 879–893.

> **We present a characterization of a $C_0$-semigroup $\{T(t)\}_{t \geq 0}$ which is an $m$-isometry, for every $t$. Indeed, $\{T(t)\}_{t \geq 0}$ is an $m$-isometry for any $t$ if and only if the mapping $t \in \mathbb{R}^+ \to \|T(t)x\|^2$ is a polynomial of degree less than $m$ for each $x \in H$. This result is used to study $m$-isometric right translation semigroup on weighted $L_p$-spaces. Moreover, we prove that a non-unitary 2-isometry on a Hilbert space satisfying the kernel condition, that is, $T^*T(\ker T^*) \subset \ker T^*$, then $T$ can be embedded into a $C_0$-semigroup if and only if $dim(\ker T^*) = \infty$.**

[D] T. Bermúdez, A. Martinón and H. Zaway, *Some examples of $m$-isometries*, submitted to publication.

> **We give examples of $m$-isometries on $n$-finite dimensional Hilbert spaces. Indeed, we prove that any strict $(2n-1)$-isometry on $\mathbb{C}^n$ is written in the following form $\lambda I + Q$, where $Q$ is a nilpotent operator and $|\lambda| = 1$. Also, we construct an $(m+1)$-isometry with an $m$-isometry given. Besides, we identify how linear $m$-isometry changes volumes.**