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Titre : Homogenization theory for the derivation of high-order macroscopic models of diffusion MRI

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Abstract

Diffusion Magnetic Resonance Imaging (dMRI) is a novel non-invasive imaging technique which is based on exploiting the displacement of water molecules in biological tissues and particularly in the human brain to measure diffusion in those tissues. This imaging modality is becoming very useful in recent years. Some applications of diffusion imaging are now validated for use in routine clinical tests (cerebral ischemia, abscess versus necrotic tumor, ...). Other applications are still being evaluated (inflammatory pathology ...). To encode the diffusion of water molecules, dMRI is based on applying two opposite gradients of important and equal intensity in a particular direction.

Our objective in this these is to provide simplified macroscopic models that offer a useful tool for simulating and interpreting dMRI images. More precisely, we want to build models that can be used to obtain quantitative informations of the cell's structures (volume of extra- and -intra- cellular, membrane permeability, cells orientations, ...) from dMRI signals.

We use homogenization theory to establish a new macroscopic model for the complex transverse water proton magnetization in a voxel due to diffusion-encoding magnetic field gradient pulses in the case of biological tissue with impermeable membranes. In this model, new higher-order diffusion tensors emerge and offer more information about the structure of the biological tissues. We explicitly solve the macroscopic model to obtain an ordinary differential equation for the diffusion MRI signal that has similar structure as diffusional kurtosis imaging models. We finally present some validating numerical results on synthetic examples showing the accuracy of the model with respect to signals obtained by solving the Bloch-Torrey equation.

Then, with the same periodic homogenised technique used in the case of biological tissue with impermeable membranes, we search a higher-order macroscopic homogenized model for the complex transverse magnetization in the case of permeable membranes. We take particular choices for the scaling for the membrane permeability which have an influence on the form of the obtained macroscopic model. We derive the ODE model that corresponds to each obtained macroscopic model. We present several numerical simulations in which we prove the convergence of the new approximate signals and we compare them with the reference Bloch-Torrey signal in terms of the box size.

In a simple two dimensional cellular configuration, we analyze the approximate signals and we compare them with the reference signal of Bloch-Torrey equation. Using physical realistic parameters, we vary the diffusion time, the cell permeability and the box size. These tests allow us to check the effectiveness of the approximate dMRI signals and to explore regimes where the high-order models perform better than the zero- and first-order models.