# Weighted Stepanov-like pseudo almost automorphic solutions of class $r$ for some partial differential equations


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# Introduction

In this work, we study the existence and uniqueness of Stepanov-like pseudo almost automorphic solutions of class $r$ for the following neutral partial functional differential equation

$$u'(t) = Au(t) + L(u_t) + f(t) \text{ for } t \in \mathbb{R}, \tag{1.1}$$

- $A$ is a linear operator on a Banach space $X$ satisfying the Hille-Yosida condition
- $L$ is a bounded linear operator from $C$ into $X$
- $f : \mathbb{R} \to X$ is continuous
- $u_t$ denotes the history function of $C$ defined by
$$u_t(\theta) = u(t+\theta) \text{ pour } -r \leq \theta \leq 0.$$

# Properties of $(\mu,\nu)$-Stepanov-like pseudo almost automorphic functions of class $r$

## Definition

The Bochner transform $f^b(t,s),\ t\in\mathbb{R},\ s\in[0,1]$, of a function $f(t)$ on $\mathbb{R}$, with values in $X$, is defined by

$$f^b(t,s)=f(t+s).$$

Let $p\in[1,+\infty[$. The space $BS^p(\mathbb{R},X)$ of all Stepanov bounded functions, with the exponent $p$, consists of all measurable functions $F$ on $\mathbb{R}$ with values in $X$ such that $f^b\in L^\infty(\mathbb{R},L^p([0,1],X))$. This space is a Banach space with the norm

$$||f||_{S^p}=\sup_{t\in\mathbb{R}}\left(\int_t^{t+1}|f(s)|^p ds\right)^{\frac{1}{p}}.$$

# Properties of $(\mu,\nu)$-Stepanov-like pseudo almost automorphic functions of class $r$

## Definition

A function $f \in BS^p(\mathbb{R}, X)$ is called $(\mu,\nu)$-$S^p$-pseudo-almost automorphic of class $r$ (or Stepanov-like pseudo-almost automorphic of class $r$) if it can be expressed as $f = h + \varphi$, where $h^b \in AA(\mathbb{R}, L^p((0,1), X))$ and $\varphi^b \in \mathscr{E}(\mathbb{R}, L^p((0,1), X), \mu, \nu, r)$ i.e.,

$$\lim_{\tau \to +\infty} \frac{1}{\nu[-\tau,\tau]} \int_{-\tau}^{\tau} \sup_{\theta \in [t-r,t]} \left( \int_{\theta}^{\theta+1} |\varphi(s)|^p ds \right)^{\frac{1}{p}} d\mu(t) = 0.$$

The collection of such functions will be denoted by $PAAS^p(\mathbb{R}, X, \mu, \nu, r)$.

# Properties of $(\mu,\nu)$-Stepanov-like pseudo almost automorphic functions of class $r$

In other words, a function $f \in L^p(\mathbb{R}, X)$ is said to be $S^p$-pseudo-almost automorphic of class $r$ if its Bochner transform $f^b : \mathbb{R} \to L^p(0,1), X)$ is pseudo-almost automorphic of class $r$ in the sense for each real sequence $(s_m)$, there exists a subsequence $(s_n)$ and a function $g \in L^p_{loc}(\mathbb{R}; X)$ such that

$$\lim_{n\to\infty} \left( \int_t^{t+1} |g(s) - h(s+s_n)|^p ds \right)^{\frac{1}{p}} = 0$$

and

$$\lim_{n\to+\infty} \left( \int_t^{t+1} |g(s+s_n) - h(s_n)|^p ds \right)^{\frac{1}{p}} = 0$$

pointwise on $\mathbb{R}$ and

$$\lim_{\tau\to+\infty} \frac{1}{\nu[-\tau,\tau]} \int_{-\tau}^{\tau} \sup_{\theta\in[t-r,t]} \left( \int_\theta^{\theta+1} |\varphi(s)|^p ds \right)^{\frac{1}{p}} d\mu(t) = 0.$$

# Properties of $(\mu,\nu)$-Stepanov-like pseudo almost automorphic functions of class $r$

From $\mu,\nu \in \mathcal{M}$, we formulate the following hypothesis.

($\mathbf{H_1}$) Let $\mu,\nu \in \mathcal{M}$ be such that $\limsup_{\tau\to+\infty} \dfrac{\mu([-\tau,\tau])}{\nu([-\tau,\tau])} = \alpha < \infty$.

## Proposition

Assume that ($\mathbf{H_1}$) holds. The space $\mathscr{E}(\mathbb{R}; L^p((0,1),X),\mu,\nu,r)$ endowed with the $\|\,.\,\|_{S^p}$ norm is a Banach space.

# Properties of $(\mu,\nu)$-Stepanov-like pseudo almost automorphic functions of class $r$

## Proposition

Assume that $(\mathbf{H_1})$ holds and let $\mu,\nu \in \mathcal{M}$ and I be a bounded interval (eventually $I = \varnothing$). The following are equivalent

$i)$ $f \in \mathcal{E}(\mathbb{R}; L^p((0,1),X),\mu,\nu,r)$.

$ii)$ $$\lim_{\tau\to+\infty} \frac{1}{\nu([-\tau,\tau]\setminus I)} \int_{[-\tau,\tau]\setminus I} \sup_{\theta\in[t-r,t]} \left(\int_{\theta}^{\theta+1} |f(s)|^p ds\right)^{\frac{1}{p}} d\mu(t) = 0.$$

$iii)$ For any

$$\varepsilon > 0,\ \lim_{\tau\to+\infty} \frac{\mu\left(\left\{t \in [-\tau,\tau]\setminus I : \sup_{\theta\in[t-r,t]} \left(\int_{\theta}^{\theta+1} |f(s)|^p ds\right)^{\frac{1}{p}} > \varepsilon\right\}\right)}{\nu([-\tau,\tau]\setminus I)} = 0.$$

# Weighted Stepanov like pseudo almost automorphic solutions of class $r$

Let us introduce the part $A_0$ of the operator $A$ in $\overline{D(A)}$ which is defined by

$$\begin{cases} D(A_0) = \{x \in D(A) : \ Ax \in \overline{D(A)}\} \\ A_0 x = Ax \text{ ; for } \ x \in D(A_0) \end{cases}$$

We make the following assumption :

$(\mathbf{H_4})$ $A$ satisfies the Hille-Yosida condition.

$(\mathbf{H_5})$ $T_0(t)$ is compact on $\overline{D(A)}$ for every $t > 0$.

# Weighted Stepanov like pseudo almost automorphic solutions of class $r$

On a

## Proposition

[a] Assume that $(\mathbf{H_4})$ and $(\mathbf{H_5})$ hold and the semigroup $(\mathcal{U}(t))_{t\geq 0}$ is hyperbolic. If $f \in BC(\mathbb{R}; X)$, then there exists a unique bounded solution $u$ of equation (1.1) on $\mathbb{R}$, given by

$$\begin{aligned} u_t &= \lim_{\lambda\to+\infty}\int_{-\infty}^{t} \mathcal{U}^s(t-s)\Pi^s(\widetilde{B}_\lambda X_0 f(s))ds \\ &+ \lim_{\lambda\to+\infty}\int_{+\infty}^{t} \mathcal{U}^u(t-s)\Pi^u(\widetilde{B}_\lambda X_0 f(s))ds \ \text{ pour } \ t \in \mathbb{R}, \end{aligned}$$

a. K. Ezzinbi, S. Fatajou, G.M. N'Guérékata, *$C^n$-almost automorphic solutions for partial neutral functional differential equations*, Applicable Analysis, 86 :9, (2007), 1127-1146.

# Weighted Stepanov like pseudo almost automorphic solutions of class $r$

where for each $t \geq 0$, $\mathcal{U}(t)$ is defined on $C_0$ by

$$\mathcal{U}(t) = v_t(.,\varphi)$$

and $v(.,\varphi)$ is the solution of the following homogeneous equation

$$\begin{cases} \dfrac{d}{dt}v(t) = Av(t) + L(v_t) \text{ for } t \geq 0 \\ \\ v_0 = \varphi \in C. \end{cases}$$

# Weighted Stepanov like pseudo almost automorphic solutions of class $r$

## Proposition

Let $h \in AA_c(\mathbb{R}; L^p((0,1),X))$, then $\Gamma h \in AA_c(\mathbb{R}; L^p((0,1),X))$, where $\Gamma$ is defined by

$$\begin{aligned}\Gamma h(t) &= \Big[\lim_{\lambda\to+\infty}\int_{-\infty}^{t}\mathcal{U}^s(t-s)\Pi^s(\widetilde{B}_\lambda X_0 h(s))ds \\ &\quad + \lim_{\lambda\to+\infty}\int_{+\infty}^{t}\mathcal{U}^u(t-s)\Pi^u(\widetilde{B}_\lambda X_0 h(s))ds\Big](0).\end{aligned}$$

# Weighted Stepanov like pseudo almost automorphic solutions of class $r$

## Theorem

Let $\mu,\nu \in \mathcal{M}$. Assume that $(\mathbf{H_2})$ holds and $g \in \mathcal{E}(\mathbb{R}; L^p((0,1),X),\mu,\nu,r)$, then $\Gamma g \in \mathcal{E}(\mathbb{R}; L^p((0,1),X),\mu,\nu,r)$.

**Proof.** For each $n = 1,2,3,\ldots$, let be $X_n$ defined by

$$\begin{aligned} X_n(t) &= \lim_{\lambda\to+\infty}\int_{t-n}^{t-n+1} \mathcal{U}^s(t-s)\Pi^s(\widetilde{B}_\lambda X_0 g(s))ds \\ &\quad - \lim_{\lambda\to+\infty}\int_{t+n-1}^{t+n} \mathcal{U}^u(t-s)\Pi^u(\widetilde{B}_\lambda X_0 g(s))ds \ \text{ for } \ t \in \mathbb{R}, \end{aligned}$$

$K = \max(\overline{M}\widetilde{M}|\Pi^s|, \overline{M}\widetilde{M}|\Pi^u|)$ and $C_q(K,\omega) = \dfrac{K}{\sqrt[q]{q\omega}}(e^{q\omega}+1)^{\frac{1}{p}} \times \displaystyle\sum_{n=1}^{+\infty} e^{-q\omega n}$.

# Weighted Stepanov like pseudo almost automorphic solutions of class $r$

We prove that $X_n \in \mathcal{E}(\mathbb{R}; L^p((0,1),X),\mu,\nu,r)$ and its uniform limit belongs $\mathcal{E}(\mathbb{R}; L^p((0,1),X),\mu,\nu,r)$. Observing that

$$\Gamma g(t) = \sum_{n=1}^{+\infty} X_n(t),$$

it follows that $\Gamma g(t) \in \mathcal{E}(\mathbb{R}; L^p((0,1),X),\mu,\nu,r)$.$\blacksquare$

For the existence of pseudo almost automorphic solution, we make the following assumption :

$(\mathbf{H_6})$ $f : \mathbb{R} \to X$ is $cl(\mu,\nu)$-$S^p$-pseudo almost automorphic of class $r$.

**Theorem**

Assume $(\mathbf{H_1})$, $(\mathbf{H_2})$, $(\mathbf{H_4})$, $(\mathbf{H_5})$ and $(\mathbf{H_6})$ hold. Then equation (1.1) has a unique $cl(\mu,\nu)$-$S^p$-pseudo almost automorphic solution of class $r$.

# Weighted Stepanov like pseudo almost automorphic solutions of class $r$

Our next objective is to show the existence of pseudo almost automorphic solutions of class $r$ for the following problem :

$$u'(t) = Au(t) + L(u_t) + f(t, u_t) \text{ for } t \in \mathbb{R} \tag{3.1}$$

where $f : \mathbb{R} \times C \to X$ is continuous.
For the sequel, we make the following assertion. :

**($\mathbf{H_7}$)** $f : \mathbb{R} \times C \to X$ is uniformly pseudo compact almost automorphic such that there exists a function $L_f \in L^p(\mathbb{R}, \mathbb{R}^+)$ such that

$$|f(t,\varphi_1) - f(t,\varphi_2)| \leq L_f(t)\|\varphi_1 - \varphi_2\|_{S^p}, \text{ for all } t \in \mathbb{R}, \ \varphi_1, \varphi_2 \in BS^p([-r, 0]$$

where $L_f$ satisfies the results concerning the composition of $\mu$-pseudo almost automorphic functions.

**($\mathbf{H_8}$)** The instable space $U \equiv \{0\}$.

# Weighted Stepanov like pseudo almost automorphic solutions of class $r$

## Theorem

Assume $(\mathbf{H_1})$, $(\mathbf{H_2})$, $(\mathbf{H_4})$, $(\mathbf{H_5})$, $(\mathbf{H_7})$ and $(\mathbf{H_8})$ hold. Then equation (3.1) has a unique $cl(\mu,\nu)$-$S^p$-pseudo compact almost automorphic mild solution of class $r$.

**Idea of proof** Since the unstable space $U \equiv \{0\}$, then $\Pi^u \equiv 0$.Consider now the mapping

$$\mathcal{H} : PAA_c(\mathbb{R}; L^p((0,1),X),\mu,\nu,r) \to PAA_c(\mathbb{R}; L^p((0,1),X),\mu,\nu,r)$$

defined for $t \in \mathbb{R}$ by

# Weighted Stepanov like pseudo almost automorphic solutions of class $r$

$$(\mathcal{H}x)(t) = \Big[\lim_{\lambda\to+\infty}\int_{-\infty}^{t}\mathcal{U}^s(t-s)\Pi^s(\widetilde{B}_\lambda X_0 f(s,x_s))ds\Big](0).$$

**Case 1 :** $L_f \in L^1(\mathbb{R})$ :

Let $x_1, x_2 \in PAA_c(\mathbb{R}; L^p((0,1),X),\mu,\nu,r)$, then

$$\| \mathcal{H}^n x_1 - \mathcal{H}^n x_2\|_{S^p} \leq \frac{(\overline{M}\widetilde{M}|\Pi^s|\ |L_f|_{L^1(\mathbb{R})})^n}{n!}\|x_1 - x_2\|_{S^p}.$$

Let $n_0$ be such that $\dfrac{(\overline{M}\widetilde{M}|\Pi^s|\ |L_f|_{L^1(\mathbb{R})})^{n_0}}{n_0!} < 1$. By Banach fix point Theorem, $\mathcal{H}$ has a unique point fixed and this fixed point satisfies the integral equation

# Weighted Stepanov like pseudo almost automorphic solutions of class $r$

$$u_t = \lim_{\lambda \to +\infty} \int_{-\infty}^{t} \mathcal{U}^s(t-s)\Pi^s(\widetilde{B}_\lambda X_0 f(s,u_s))ds.$$

**Case 2 :** $L_f \in L^p(\mathbb{R})$; $(1 < p < \infty)$ :

First, put

$$\mu(t) = \int_{-\infty}^{t} (L_f(s))^p ds.$$

Then we define an equivalent norm over $PAAS^p(\mathbb{R}, X)$ as follows,

$$|f|_c = \sup_{t\in\mathbb{R}} e^{-c\mu(t)} \left( \int_t^{t+1} |f(s)|^p \right)^{\frac{1}{p}},$$

where $c$ is a fixed positive number.

# Weighted Stepanov like pseudo almost automorphic solutions of class $r$

Then

$$|\mathcal{H}x_1(t) - \mathcal{H}x_2(t)|_c \leq \frac{\overline{M}\widetilde{M}|\Pi^s|}{(pc)^{\frac{1}{p}} \times (\omega q)^{\frac{1}{q}}} |x_1 - x_2|_c.$$

Fix $c > 0$ so large, then the function $c \mapsto \dfrac{1}{(pc)^{\frac{1}{p}}}$ converges to 0 when $c$ tends to $+\infty$. It follows that for $c > 0$ so large enough we have $\dfrac{\overline{M}\widetilde{M}|\Pi^s|}{(pc)^{\frac{1}{p}} \times (\omega q)^{\frac{1}{q}}} < 1$. Thus $\mathcal{H}$ is a contractive mapping. We conclude that equation(3.1) has one and only one $cl(\mu,\nu)$-$S^p$-pseudo almost automorphic solution of class $r$ which ends the proof.■

# Weighted Stepanov like pseudo almost automorphic solutions of class $r$

## Proposition

Assume that $(\mathbf{H_1})$, $(\mathbf{H_3})$, $(\mathbf{H_4})$, $(\mathbf{H_5})$, $(\mathbf{H_8})$ hold and $f$ is Lipschitzian with respect the second argument. If

$$Lip(f) < \frac{\omega}{\overline{M}\widetilde{M}|\Pi^s|},$$

then equation (3.1) has a unique $cl(\mu)$-pseudo almost automorphic solution of class $r$, where $Lip(f)$ is the Lipschitz constant of $f$.

# Weighted Stepanov like pseudo almost automorphic solutions of class $r$

**Sketch of proof.** Let us set $k = Lip(f)$, we have

$$|\mathcal{H}x_1(t) - \mathcal{H}x_2(t)| \leq \frac{|\Pi^s|\overline{M}\widetilde{M}k\|x_1 - x_2\|_{S^p}}{\omega}.$$

Consequently $\mathcal{H}$ is a strict contraction if $k < \dfrac{\omega}{\overline{M}\widetilde{M}|\Pi^s|}$. ■

# Application

For illustration, we propose to study the existence of solutions for the following model

$$
\begin{cases} \dfrac{\partial}{\partial t}z(t,x) = \dfrac{\partial^2}{\partial x^2}z(t,x) + \displaystyle\int_{-r}^{0} G(\theta)z(t+\theta,x))d\theta \\ \qquad + \sin\left(\dfrac{1}{2+\cos t+\cos\sqrt{2}t}\right) + g(t) \\ \qquad\qquad + \displaystyle\int_{-r}^{t} e^{\omega(-t+\theta)}h(\theta,z(t+\theta,x))d\theta \text{ for } t\in\mathbb{R} \text{ and } x\in[0,\pi] \\ z(t,0)=z(t,\pi)=0 \text{ for } t\in\mathbb{R}, \end{cases} \tag{4.1}
$$

where $r \geq 0$, $G : [-r,0] \to \mathbb{R}$ is a continuous function, $h : [-r,0]\times\mathbb{R} \to \mathbb{R}$ is continuous and lipschitz continuous with respect to the second argument, $\omega$ is a positive real number and $g : \mathbb{R}\times[0,\pi] \to \mathbb{R}$ is a bounded continuous function defined by

# Application

$$g(t) = \begin{cases} 0 \text{ for } t \leq 0 \\ te^{-t} \text{ for } t \geq 0. \end{cases}$$

For example, we can take $G(\theta) = \dfrac{\theta^2 - 1}{(\theta^2 + 1)^2}$ for and $\theta \in [-r, 0]$ and $h(\theta, x) = \theta^2 + \sin\left(\dfrac{x}{2}\right)$ for $(\theta, x) \in [-r, 0] \times \mathbb{R}$. We can see that $G : [-r, 0] \to \mathbb{R}$ is a continuous function and $|h(\theta, x_1) - h(\theta, x_2)| \leq \dfrac{1}{2}|x_1 - x_2|$, which implies $h : [-r, 0] \times \mathbb{R} \to \mathbb{R}$ is continuous and lipschitz continuous with respect to the second argument.

To rewrite equation (4.1) in the abstract form, we introduce the space $X = C_0([0, \pi]; \mathbb{R})$ of continuous function from $[0, \pi]$ to $\mathbb{R}^+$ equipped with the uniform norm topology.

# Application

Soit $A : X \to X$ défini par

$$\begin{cases} D(A) = \{y \in X : y \in C^2([0,\pi],\mathbb{R}) \text{ et } y', y'' \in X\} \\ Ay = y''. \end{cases}$$

Let $A : D(A) \to X$ be defined by

$$\begin{cases} D(A) = \{y \in X \cap C^2([0,\pi],\mathbb{R}) : \; y'' \in X\} \\ Ay = y''. \end{cases}$$

Then $A$ satisfied the Hille-Yosida condition in $X$. Moreover the part $A_0$ of $A$ in $\overline{D(A)}$ is the generator of strongly continuous compact semigroup $(T_0(t))_{t\geq 0}$ on $\overline{D(A)}$. It follows that $(\mathbf{H_4})$ and $(\mathbf{H_5})$ are satisfied.

# Application

We define $f : \mathbb{R} \times C \to X$ and $L : C \to X$ as follows

$$\begin{aligned} f(t,\varphi)(x) &= \sin\left(\frac{1}{2+\cos t+\cos\sqrt{2}t}\right)+g(t) \\ &\quad + \int_{-r}^{t} e^{\omega(-t+\theta)} h(\theta,\varphi(\theta)(x))d\theta \ \text{ for } x \in [0,\pi] \ \text{ and } \ t \in \mathbb{R}, \\ L(\varphi)(x) &= \int_{-r}^{0} G(\theta)\varphi(\theta)(x))d\theta \ \text{ for } \ -r \leq \theta \leq 0 \ \text{ and } \ x \in [0,\pi]. \end{aligned}$$

Let us pose $v(t) = z(t,x)$. Then system (4.1) takes the following abstract form

$$v'(t) = Av(t) + L(v_t) + f(t, v_t) \text{ for } t \in \mathbb{R}. \tag{4.2}$$

Consider the measures $\mu$ and $\nu$ where its Radon-Nikodym derivative are respectively $\rho_1, \rho_2 : \mathbb{R} \to \mathbb{R}$ defined by

# Application

$$\rho_1(t) = \begin{cases} 1 \ \text{ for } \ t > 0 \\ \\ e^t \ \text{ for } \ t \leq 0. \end{cases}$$

and

$$\rho_2(t) = |t| \ \text{ for } \ t \in \mathbb{R}$$

i.e $d\mu(t) = \rho_1(t)dt$ and $d\nu(t) = \rho_2(t)dt$ where $dt$ denotes the Lebesgue measure on $\mathbb{R}$ and

$$\mu(A) = \int_A \rho_1(t)dt \ \text{ for } \ \nu(A) = \int_A \rho_2(t)dt \ \text{ for } \ A \in \mathscr{B}.$$

Then $\mu, \nu \in \mathcal{M}$, $\mu, \nu$ satisfy Hypothesis $(\mathbf{H_3})$ and $\sin\left(\dfrac{1}{2+\cos t + \cos\sqrt{2}t}\right)$ is compact almost automorphic.

# Application

Moreover

$$\limsup_{\tau\to+\infty}\frac{\mu([-\tau,\tau])}{\nu([-\tau,\tau])}=\limsup_{\tau\to+\infty}\frac{1-e^{-\tau}+\tau}{\tau^2}=0<\infty,$$

which implies that $(\mathbf{H_1})$ is satisfied.

Let $p\geq 1$, since $r$ is given then we have

$$\lim_{\tau\to+\infty}\frac{1}{\nu([-\tau,\tau])}\int_{-\tau}^{+\tau}\sup_{\theta\in[t-r,t]}\left(\int_{\theta}^{\theta+1}|g(s)|^p ds\right)^{\frac{1}{p}}dt=0.$$

It follows that $g\in\mathcal{E}(\mathbb{R};L^p((0,1),X),\mu,\nu,r)$.

# Application

For every $\varphi_1, \varphi_2 \in BS^p(\mathbb{R}; X))$ and $t \geq 0$, we have

$$|f(t,\varphi_1) - f(t,\varphi_2)| \leq \frac{C_q(\omega)}{2} \sup_{0 \leq x \leq \pi} \|\varphi_1(x) - \varphi_2(x)\|_{S^p}.$$

Consequently, we conclude that $f$ is Lipschitz continuous and $cl(\mu,\nu)$-$S^p$-pseudo almost periodic of class $r$. Moreover, $L$ is a bounded linear operator from $C$ to $X$.

# Application

We assume that :

$$(\mathbf{H_9}) \int_{-r}^{0} |G(\theta)| d\theta < 1.$$

then the semigroup $(\mathcal{U}(t))_{t\geq 0}$ is hyperbolic .
Observe that

$$\int_{-r}^{0} |G(\theta)| d\theta = \frac{r}{r^2+1} < 1 \text{ if } r < 1$$

and

$$\int_{-r}^{0} |G(\theta)| d\theta = 1 - \frac{r}{r^2+1} < 1 \text{ if } r \geq 1,$$

then $(\mathbf{H_9})$ is satisfied and the instable space $U \equiv \{0\}$.

# Application

We deduce the following result.

## Proposition

Under the above assumptions, if $Lip(h) = \frac{1}{2} < \frac{1}{C_q(\omega)}$, i.e $C_q(\omega) < 2$, then equation (4.2) has a unique $cl(\mu, \nu)$-$S^p$-pseudo almost automorphic solution $v$ of class $r$.

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# Merci pour votre aimable attention